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ABSTRACT

This study investigated manifestations of collaborative work with pairs of kindergarten children during counting tasks designed to promote early number development. The data were collected by means of classroom observations, interviews with individual students, and teaching sessions with pairs of students. Ten students were paired to be compatible with respect to their counting stage. During the teaching sessions and the interviews, the students encountered various counting tasks and games incorporating counting. As the pairs participated in the teaching sessions, their actions were documented on video- and audiotape. Analyses indicated that the students generated four strategies to organize their counting: (1) counting side by side; (2) counting at the same time; (3) taking turns; and (4) working cooperatively. Whereas students counted independently with the strategies of counting side by side or at the same time, they coordinated their actions in order to decide who would start and who would wait in taking turns. The waiting partner engaged in several activities such as watching, watching and joining, solving the partner's task, supporting, helping, commenting, correcting, and completing the partner's task. There were two favorable conditions for the initiation of a cooperative counting episode: (1) counting tasks with more than 10 items that the students could not solve alone; and (2) the teacher's explicit instruction to work as a team. Cooperative strategies were sharing resources, double counting, delegating tasks, and counting alternately. Results of the study suggest that working in pairs can be valuable for kindergarten students in terms of enhancing the students' cognitive development and promoting sophisticated ways of social interaction. (Contains 39 references.) (WP)

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Organization of Counting in Joint Counting Tasks and the Emergence of Cooperation in Pairs of Kindergarten Students

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Abstract

The purpose of this study was to investigate and document possibilities for and manifestations of collaborative work with pairs of kindergarten students during counting tasks designed to promote early number development.

Ten students were paired to be compatible with respect to their counting stage. For a period of 4 months, each pair was taught once a week for 30 to 45 minutes. The students were addressed as a pair and provided with only one set of counting materials (joint counting tasks). In addition to counting tasks, the students encountered tasks addressing the order of the number word sequence, tasks focusing on visual and auditory patterns, and games incorporating counting and patterns.

The students generated four strategies to organize their counting: counting side by side, counting at the same time, taking turns, and working cooperatively. When counting side by side or at the same time, the students were engaged in independent counting activities with no or little attention to the partner. When taking turns, the students had to coordinate their actions by agreeing on who was to start and who would wait. Productive activities of the waiting partner included: watching, watching and joining, solving the partner's task, supporting, helping, commenting, correcting, and completing the partner's task. The most advanced students, who had constructed or were close to constructing the initial number sequence, engaged in cooperative counting episodes.

A cooperative counting episode was defined as a counting activity in which the students' individual counting acts merged into a single counting activity. This counting activity was preceded by a moment of planning in which a student formulated a goal and projected the initial counting acts. Two contexts provided favorable conditions for the initiation of a cooperative counting episode: counting tasks with more than 10 items covered which the students--at the time--could not solve alone, and the teacher's explicit instruction to work "as a team." Cooperative strategies included: sharing resources, double counting, delegating tasks, and counting alternately.

INDEX WORDS: Kindergarten, Development of Counting, Social Interaction, Cooperation

Within an apparent tension between cooperation and competition (Johnson & Johnson, 1991; May & Doob, 1937; National Research Council, 1989), learning in small cooperative groups has received ever growing attention. Cooperative goal structures and learning situations--in comparison to individualistic and competitive arrangements--have been studied and described extensively (e.g., Davidson, 1990; Deutsch, 1949; Johnson & Johnson, 1991; Johnson, Johnson, Holubec, & Roy, 1984; Kamii & DeClark, 1985; Kamii & DeVries, 1980; Kamii & Joseph, 1989; Slavin, 1983, 1988, 1990, 1991; Wood, Cobb, & Yackel, 1990; Wood & Yackel, 1990; Yackel, Cobb, & Wood, 1991). As Good, Mulryan, and McCaslin (1992) argued, however, the majority of studies have been outcome studies focusing on academic (i.e., achievement) and affective outcomes under cooperative, competitive, and individualistic learning conditions; not enough attention has been paid to the group processes. The authors referred to Bossert's (1988) assertion that many researchers of cooperative learning have taken a black box approach: "Students are assigned to one of two treatments . . . , outcome measures are collected, and one method is compared to the other. When effects are found, post hoc rationales explain the results" (Bossert, 1988, p. 233). Good et al. (1992) agreed with Bossert that researchers should not make guesses about possible mediating processes. Instead:

> Mediating factors that explain why cooperative procedures work must be examined in observational research, and assumptions about desirable learning processes during small-group work must be verified and modified on the basis of such research. (Good et al., 1992, p. 176)

Davidson and Kroll (1991) addressed the same issue:

> An additional important question to consider is just exactly what goes on during various types of cooperative learning. To date, a relatively small percent of the studies have attempted to study the interactions that take place during cooperative work to determine how various academic, social, or psychological effects are produced. (p. 363)

Good et al. (1992) emphasized the need for studies that explore different variables and combinations of those variables to arrive at theories "about what group processes facilitate various learner outcomes" (p. 182). Among the individual and group

variables that merit further study are age and grade. Good et al. asked whether students in primary grades can be expected to work cooperatively to the same degree and in the same way as older students (p. 185). Attention span, energy for academic work, and interest in peer work vary with age and need to be addressed accordingly. In addition, young students might not understand the role shift of the teacher between whole-class and small-group instruction; they might not understand that they have to learn from peers as well as from the teacher.

Collaborative work among kindergarten and first-grade students has not received as much attention as cooperation among older students. Slavin (1991), for example, noted that "cooperative learning has been used--and investigated--in every imaginable subject in grades 2-12" (p. 72); he mentioned no studies with younger students. Similarly, the overviews of Webb (1991) and Slavin (1983, 1990) made no reference to studies with students younger than second grade, and the overview of Johnson and Johnson (1985) contained only a few studies that included first-grade students. Totten and his collaborators' annotated bibliography (Totten, Sills, Digby, & Russ, 1991) also included only a few references to young children, among them Pepitone and Vanderbilt's (1980) work on sharing between kindergarten students and Azmitia's (1988) investigation of solitary and collaborative performances of preschool children in a construction task. Azmitia's study was an experimental, outcome-oriented study that linked working condition (solitary, same-ability pairs, mixed-ability pairs) and expertise (novice, expert) to achievement (building accuracy).

Like Azmitia's (1988) work, this paper focuses on collaborative work of young children. It is based on a study conducted with five pairs of kindergarten students. The study was an observational, process-oriented study exploring possibilities for and manifestations of collaborative work in tasks designed to promote early number development. Pairs were used rather than groups of three or four students primarily because the children were so young. The study traced the development of interaction and mutual coordination of actions in teaching sessions conducted over a period of 4

months. The need for some form of collaboration between the students in each pair was established through *joint counting tasks*, that is, the students were generally addressed as a pair and provided with only one set of counting materials ("resource interdependence," Johnson & Johnson, 1991, p. 135). The models for small-group instruction and cooperation developed by Cobb and collaborators (e.g., Wood, Cobb, & Yackel, 1990; Wood & Yackel, 1990; Yackel et al., 1991) and by Kamii (e.g., Kamii, 1982; Kamii & DeClark, 1985) with their focus on problem-solving, meaningful strategies and solutions, and mutual agreements and grounded in a constructivist learning theory (Yackel et al., 1991, p. 391) guided the design and conduct of this study. With respect to the development of counting, the study built on the framework developed by Steffe and his collaborators (e.g., Steffe & Cobb, 1988; Steffe, von Glasersfeld, Richards, & Cobb, 1983) and extended by Wright (1989).

I give an overview of the students' development of counting during the course of the teaching sessions. I then present the different strategies the students developed to organize their counting in joint counting tasks and elaborate on the emergence of cooperation in two of the five pairs of students. Finally, I analyze one cooperative counting episode in detail.

Method

Participants

The study was conducted with five pairs of kindergarten students in the 1990 - 1991 school year. The 10 students (five girls, five boys; three African-American, one Hispanic, six white) attended the same kindergarten class in a public elementary school in a university town in the Southeastern United States. On the basis of classroom observations and the analysis of initial interviews (January 1991), I selected the participants so that the study would include students in different counting stages (Steffe, 1992). The pairs were assembled so that the students in each pair were as

compatible as possible with respect to their development of counting. Table 1 provides an overview of the participating students, their ethnic origins, their ages at the beginning of the teaching sessions, and the number of teaching sessions the students of each pair attended. In addition, the table contains the reading group attended by each student, with group A denoting the more advanced reading group.

Table 1

Characteristics of Participating Students

Student	Ethnic origin	Age (years; months)	Reading group	Number of teaching sessions
Scott	white	5;11	B	10
Alisha	Afro-American	5;9	B	
Rita	Afro-American[a]	7;3	B	7
Charlette	Afro-American[a]	6;8	B	
Mike	white	6;3	A	11
José	Hispanic[b]	6;3	A	
Ben	white	6;2	A	11
Adam	white	6;5	A	
Emily	white[c]	6;0	A	11
Andrea	white[c]	6;4	A	

Notes: Brackets show pairs; all names are pseudonyms.
[a] Repeated kindergarten in 1990-1991; [b] bilingual;
[c] Participated in the program for gifted students in 1990-1991.

Data Collection

The data collection lasted from 26 November 1990 to 2 May 1991 and consisted of three main activities: (a) classroom observations, (b) interviews with individual

students, and (c) teaching sessions with pairs of students. The work with the students was supplemented by talks with the classroom teacher and the aide and by meetings with some of the parents. Typically, each pair was taught once a week for 30 to 45 minutes. In addition, each student participated in at least three individual sessions: the initial interview, an interview before spring holidays, and the final interview. Additional individual sessions occurred if one of the students in a pair was absent.

During the teaching sessions and the interviews, the students encountered a variety of counting tasks: tasks involving unscreened or partly screened collections, tasks involving two covered collections, missing-item tasks (Steffe & Cobb, 1988; Steffe, von Glasersfeld, Richards, & Cobb, 1983; Wright, 1989), and games that incorporated counting (Baratta-Lorton, 1976; Hughes, 1986; Kamii & DeClark, 1985; Kamii & Joseph, 1989). The counting tasks were supplemented by tasks addressing the order of the number word sequence and by tasks focusing on visual and auditory patterns. As the pairs participated in the teaching sessions, the students' further development of counting, their interactions, and their increasing ability to coordinate their own actions with those of the partner were documented on video- and audiotape.

Analysis

After each teaching session, I wrote a preliminary protocol based on at least two viewings of the videotape. After completion of all teaching sessions (May 1991), I viewed all videotapes again, in chronological order for each pair of students, and I updated and extended the protocols. The process of viewing tape segments and of refining the protocols continued throughout the data analysis.

The students' actions as seen on the videotapes and documented in the protocols were analyzed with respect to two aspects: (a) the students' development of counting, and (b) the students' organization of counting and social interaction. Based on Steffe and his associates' work (Steffe, 1992; Steffe & Cobb, 1988; Steffe et al., 1983), I developed the following behavioral indicators to classify the students:

Perceptual counting stage: The student can count perceptual collections and can enact spatial patterns.

Transition from perceptual to figurative counting stage: The student can make intuitive extensions in some contexts and can experience conflict when he or she is unable to keep track of counting acts.

Figurative counting stage: The student can make intuitive extensions in all contexts; he or she often keeps track of counting acts with finger patterns. The student can focus on the screened items to find how many are covered.

Transition from figurative counting stage to initial number sequence: The student can count on in some contexts and starts to monitor counting during the activity itself.

Initial number sequence: The student can count on in all contexts.

Implicitly nested number sequence: The student can keep track of his or her counting acts by double counting.

Explicitly nested number sequence: The student can use strategies.

With respect to the students' organization of counting and their interactions, the data were analyzed using analytic induction (Goetz & LeCompte, 1984). Analytic induction involves "scanning the data for categories of phenomena and for relationships among such categories" (pp. 179-180); working typologies and hypotheses are developed by examining initial cases and are modified and refined on the basis of subsequent cases. In a first step, I identified and coded all incidents of interaction in the first three sessions with three pairs of students, each pair representing a different counting stage. The codes described the overall framing activity (e.g. counting, talk, exploration, demonstration, game), the direction of the interaction (student-student, student-teacher, and teacher-student), and the nature of the interaction (e.g. watch, show, explain, interrupt, retreat). I organized the codes--each on a 3 x 5 in. index card--in nine groups with the pairs of students and the direction of interaction as main organizing principles. I arranged the cards to show subgroups of codes referring to the same context and identified relationships between codes attached to the same text segment, for example, "take charge--material/action" or "side by side--no conflict/conflict." I completed and refined the codes and

reorganized them as I examined more sessions. Finally, I identified changes in the students' organization of counting and their working relationships. I used the text analysis program The Ethnograph to assist in identifying, delimiting, and sorting relevant text segments and in tracing the changes in the students' organization of counting and their interactions.

Results

Development of Counting During the Teaching Sessions

At the beginning of the teaching sessions, four students were in the perceptual counting stage, four students were in transition from the perceptual to the figurative stage, one student was in the figurative stage, and one student had constructed the initial number sequence. At the end of the sessions, all students but one had progressed beyond their initial counting stage. Three students were now in a transitional phase between the perceptual and the figurative counting stage, one student had progressed to the figurative counting stage, and three students were in transition from the figurative stage to the initial number sequence; one student had constructed the initial number sequence, and one student had constructed the explicitly nested number sequence. The students' progress is summarized in Table 2.

Organization of Counting in Joint Counting Tasks

A joint counting task was defined as a task in which the students were addressed as a pair and provided with only one set of counting materials. To complete a joint counting task, the students had to come to some kind of agreement--implicit or explicit--on how to organize their counting. During the course of the teaching sessions, the students generated four strategies to organize their counting in joint counting tasks: (a) counting side by side, (b) counting at the same time, (c) taking turns, and (d) working cooperatively.

Table 2

Students' Counting Stages at the Beginning and End of the Teaching Sessions

Student	Counting stage	
	Beginning of teaching sessions January 1991	End of teaching sessions May 1991
Scott	Perceptual	Transition - perceptual to figurative
Alisha	Perceptual	Transition - perceptual to figurative
Rita	Perceptual	Transition - perceptual to figurative
Charlette	Perceptual	Perceptual
Mike	Transition - perceptual to figurative	Figurative
José	Transition - perceptual to figurative	Transition - figurative to INS[a]
Ben	Transition - perceptual to figurative	Transition - figurative to INS
Adam	Transition - perceptual to figurative	Transition - figurative to INS
Emily	Figurative	Initial number sequence
Andrea	Initial number sequence	Explicitly nested number sequence

[a] initial number sequence

Counting Side by Side

A counting activity was classified as counting side by side if each student worked by him- or herself, without much regard to the partner. Given, for example, a collection of blocks, each student moved blocks from the initial location to his or her place, uttering, "One, two, . . . ," until the original collection was exhausted. The outcome of this strategy was two or more smaller subcollections, each of which had been counted by one student, but not by the other (see Figure 1). Counting side by side occurred only in the context of counting unscreened collections and mainly at the beginning of

the teaching sessions. During the first two sessions, the students of all five pairs counted side by side at one time or another; at the end of the teaching sessions, the students of only one pair, Rita and Charlette, still solved a counting task by counting side by side.

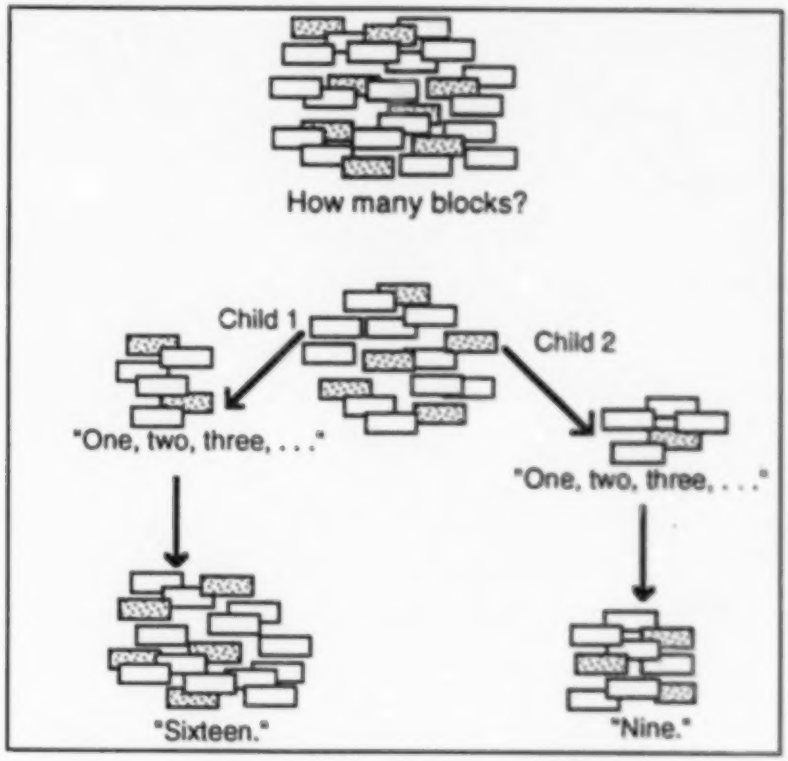

Figure 1. Counting side by side.

Counting at the Same Time

Counting at the same time differed from counting side by side in that the students did not create smaller subcollections, but each student counted all (or most of) the objects in the original collection. The students were still engaged in two separate, mostly independent, counting activities, but they pushed the objects to a common location, moved them only slightly within the vicinity of the original locations, or did not displace them at all. The students of one pair, Rita and Charlette, usually adjusted to the same rhythm both in saying the number words and in moving the blocks. The process of counting an unscreened collection at the same time is outlined in Figure 2.

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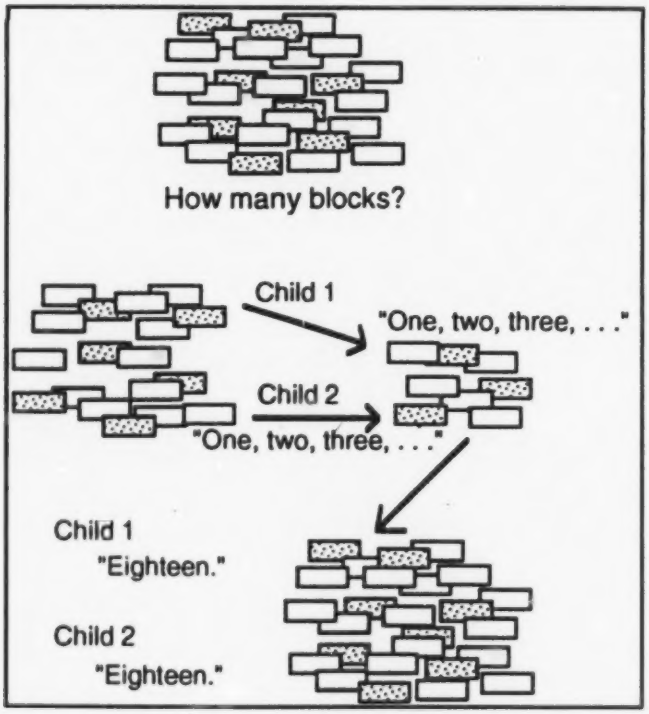

Figure 2. Counting at the same time.

In contrast to counting side by side which occurred only during work with unscreened collections, students counted at the same time in tasks involving unscreened collections and in tasks involving partly screened collections. Working with partly screened collections, students counted at the same time if one or both of the partners did not need to manipulate the objects in order to count them. Generally, only the more advanced students solved tasks involving partly screened collections at the same time. Andrea, for example, having constructed the initial number sequence, could count on and thus frequently did not need to handle the objects; uttering a number word took the place of counting the visible objects of a collection. She then kept track of her subsequent counting acts--counting the screened objects--by extending fingers until she had completed the finger pattern signifying the number of screened items. In contrast, the students in the perceptual or figurative stage needed

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to handle the materials; they usually had to point to or touch the objects in order to count them. They rarely counted at the same time but usually took turns.

Taking Turns

Taking turns was overall the most frequently used organization of counting. When taking turns, only one student at a time was involved with the objects while the partner waited (see Figure 3).

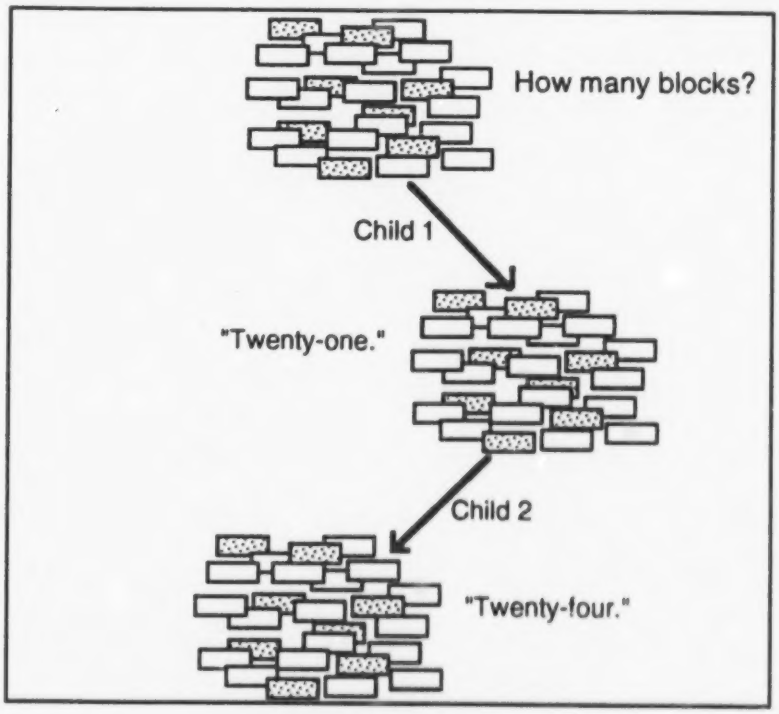

Figure 3. Taking turns.

The activity of the partner waiting for his or her turn ranged from deliberate disturbance (Mike and José) and nonparticipation (all students at one time or another) to different ways of participation: watching, watching and joining, and solving the partner's task (at the same time). Interactive ways of participation included: supporting the partner in his or her activity, helping and correcting, commenting on the partner's actions, and completing the partner's counting activity. I elaborate on some of the constructive forms of participation.

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Supporting. Supporting was a form of participation in which a student assisted the partner but did not take part in the main activity. Scott and Alisha devised this form of participation when they were asked to count beans into small boxes with the teacher emphasizing accuracy and teamwork (Session 9). Within an overall organization of taking turns, Scott and Alisha supported each other's counting activities by supplying the partner with the beans to be counted. Thus, the counting student, instead of reaching into the container holding the beans, took the beans out of the partner's hand. In all three tasks, it was the supporting partner who initiated the collaboration. Assuming this supporting role enabled the noncounting student to be involved in the activity and thereby to fulfill the obligation to work together--without disturbing the partner's counting activity.

Watching and joining. Scott, while supplying Alisha with beans, went beyond this supporting role when he joined her counting activity by subvocally uttering the number words with her. Scott's participation is an example of an activity I identified as watching and joining. In general, watching and joining referred to a counting situation in which the watching student performed only part of each counting act. In particular, the rhythmic motor activities that aid the coordination between the creation of countable unit items and the production of number words (Steffe et al., 1983) were not part of his or her actions. Watching the partner point to or move the items while uttering number words was an opportunity for the students "to see their own actions from the outside and from the inside" (Sinclair, 1990, p. 22) and thereby possibly to become aware of the motor activities and isolate them from the otherwise integrated counting acts.

Completing partner's counting activity. Continuing and completing the partner's counting activity required that a student could make sense of the partner's actions. In addition, the student had to view the partner's and his or her own counting acts as parts of a single counting episode. I illustrate this form of participation with an episode from Scott and Alisha in the context of a game (Session 9) in which Alisha had won 12

beans. The teacher gave her 6 of them and asked, "How many more?" Scott watched as she recounted the 6 beans. When she stopped, he sequentially extended six fingers while subvocally uttering the number words from seven to twelve. He then counted the extended fingers by touching his lips and answered, "Six more." An important feature of this episode was that Scott did not have to recount the six beans on the table but was able to take Alisha's counting acts, "One, two, . . . , six," as given, substituting them for his own. Scott's completion of Alisha's counting episode was his most advanced form of participation.

Helping, correcting, and commenting. Incidents of helping, correcting, and commenting on the partner's work occurred most frequently during tasks involving unscreened collections. In one such task, designed to practice the order of the number word sequence, students labeled stickers arranged in a line with numerals (see Figure 4).

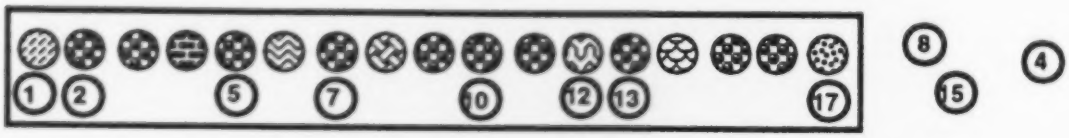

Figure 4. Labeling stickers with numerals.

Often, a student counted to find the sticker for a specific numeral. The waiting partner participated by helping ("Right here," identifies 14 for her partner; Charlette, Session 4), correcting ("One, . . . , eighteen, twenty, twenty-one, see that would be wrong, eighteen, nineteen, twenty"; Adam, Session 3), and commenting ("Her already counted this," meaning Alisha had counted an item twice; Scott, Session 1). From the beginning of the teaching sessions, this participation was more elaborate and involved than in tasks with partly screened collections, in particular for the students in the perceptual counting stage. They seemed to feel confident when counting a collection of visible objects or when identifying a particular item in a line. As observers of the

partner's counting, they could relate to his or her activity and interpret it with respect to their own experience. These interpretations seemed to be mutually compatible.

Cooperation

A joint counting activity was classified as cooperation if the counting acts of both students merged into a single counting activity with the students working toward a common goal. The merging of the students' counting acts into one activity is beautifully illustrated in the cooperative strategy counting alternately (Figure 5). The pair Andrea and Emily and the pair Ben and Adam developed this strategy toward the end of the teaching sessions when they were asked to count--as a team--beans into small boxes.

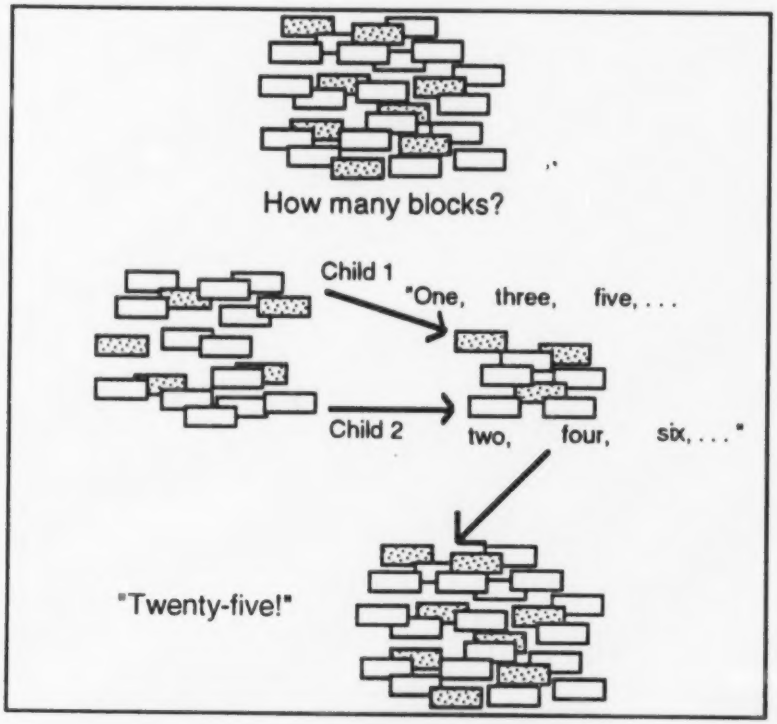

Figure 5: Counting alternately.

Many of the situations leading to a cooperative solution involved tasks in which more than 10 objects had been covered and which the students--at that time--were not able to solve alone. Only the two most advanced pairs (Andrea and Emily; Ben and

Adam) solved such counting tasks cooperatively. The two pairs developed two strategies for counting these collections: (a) sharing resources and (b) dividing the counting activity.

Sharing resources. Sharing resources refers to cooperative counting episodes in which the partners pooled their fingers in order to represent covered collections with more than 10 objects screened. The first successful completion of such a task led to a culture of cooperation in which pooling fingers became routine. Both pairs did not hesitate to use the teacher's fingers if they felt they needed them. I describe and analyze a counting episode illustrating the strategy of sharing resources later in the paper (pp. 21-25).

Dividing the counting activity. One way of dividing the counting activity was a cooperative form of double counting. In this form of cooperation, one of the partners counted the covered objects while the other kept track of how many counting acts had been performed. The following protocol illustrates how Ben's initiation of cooperation was the result of a futile attempt to keep track of his counting acts on his own. The students were trying to count how many beans were in two (closed) boxes, one containing 12 beans, the other 14. To assist in the counting process, the students used a 100-board (Kamii & DeClark, 1985; Wiegel, 1993).

> Ben: (Has his finger on the 12th field of the 100-board) Twelve--, (moves his finger to the 13th field), thirteen, (pause) that's one, thirteen--
> Fourteen, that's two, fifteen-three, sixteen--.
> (Looks up, then at Adam) You count by "one, two, three," (slides his finger along the 13th, 14th field, . . .), while I count thirteen, fourteen, all those of it (slides his finger along the fields).
> And then when we get up to fourteen, we'll see what number I get, that I'm pointed to, all right, and I'm right there.

Adam did not respond to Ben's request orally. But when Ben started to count, "Thirteen, fourteen, . . . ," sliding his fingers along the respective fields, Adam silently raised his fingers, one after the other. Adam's nonverbal response was more appropriate than a verbal counting activity might have been. The nonverbal actions allowed Ben to focus on his own counting acts and the production of the correct

number words without being distracted by a sequence of number words different from the one he was producing.

A different way of splitting a counting activity can be called start and continue. In this form of cooperative counting, one of the students starts the counting activity and counts the first part of the objects. The partner then takes over and counts the remaining objects. In start and continue, both students have to be able to relate their own counting activities to that of the partner. The student counting first has to consider the partner's counting as an extension of his or her own counting activity. The student continuing the counting episode has to be able to take the partner's counting as given and substitute it for his or her own counting activity. What distinguishes start and continue as a cooperative counting episode from mere participation in the partner's turn (e.g., completing the partner's counting, see pp. 12-13) is the act of planning and anticipation that precedes the actual activity and establishes the common goal for both students. An example of start and continue is given on p. 18.

The Emergence of Cooperation

Cooperative solutions of joint counting tasks were relatively rare events. Generally, the students preferred to work alone, either by counting at the same time or by taking turns. When counting partly screened collections or two screened collections, the students worked cooperatively only if they were unable to solve the task alone. Cooperative counting episodes involving unscreened collections evolved only as the result of specific teacher instructions to "work together, as a team." And, as mentioned before, only the two pairs with the most advanced counting schemes solved counting tasks in cooperation, whether the collections to be counted were screened or whether they were unscreened.

Cooperation as Result of Teacher Instruction

All cooperative counting episodes involving unscreened collections occurred in Session 9, toward the end of the teaching sessions, when the students were asked to

count beans into small boxes. To emphasize the necessity of working together, the teacher placed the container with the beans between the students and distributed one box at a time.

Cooperation within taking turns. Ben and Adam, like Scott and Alisha (see p. 12), organized the bean-counting task by taking turns. But within this overall organization of taking turns, they generated forms of working together that were more sophisticated than Scott and Alisha's participation through mutual support. The boys' organization and mutual involvement were the result of different ideas they pursued throughout the bean-counting tasks. Whereas Adam intended to take turns from the beginning and insisted on completing the first task alone ("Ben, I can count by myself, okay!"), Ben focused on the collaborative aspect of the task and thought of ever new ways of participating. When Adam (Task 1) bent down to collect a bean from the floor, Ben counted a few more beans into the box. Annoyed, Adam started over; now Ben held his hands beside the table in case another bean fell off. During his own turn, Ben invited Adam to participate ("You can help me. . . . Let's count"). This invitation led to a counting activity that, after some adjustment, proceeded alternately (see Figure 4). In the third task, Adam took care of Ben's desire to participate by letting him say the number words while Adam dropped the beans into the box ("You can count what I put in"). And in the last task, Ben started the counting activity and Adam completed it when Ben ran out of beans. Ben and Adam's means of mutual participation are examples of cooperation in the sense that their counting acts merged into a single counting activity and were preceded by an act of planning.

Cooperation and taking turns: A transcendent view. During the bean-counting tasks, Emily and Andrea counted alternately in the first task, took turns for the second and third task, and then decided to "do another one together." At this point, Andrea noticed that they were working in a pattern:

Andrea: I do one, you do one, we both do one.

Emily: (Joins in) We both do one, you do one, I do one.

Emily and Andrea's exchange reveals an even more sophisticated point of view than that of Ben and Adam. Whereas the boys stayed in the situation of each counting episode, Emily and Andrea were able step back from their immediate experience and look at their actions in context and as such from a different perspective. They viewed each counting episode in relation to the other episodes and evaluated their activity as a whole.

Cooperation as Result of Task Difficulty

The task format and the size of the covered collection were two factors influencing the difficulty--with respect to the students' counting stage--of a joint counting task. Tasks in an unfamiliar task format and tasks involving collections with more than 10 objects screened were problematic situations the students could not--at least not immediately--solve alone and had the potential to lead to an initiation of cooperation.

Task format as source of difficulty. The first cooperative solution of a joint counting task occurred in Session 4 in the context of the game Peek Through the Wall (Baratta-Lorton, 1976; Wiegel, 1993). Ben drew a dot-pattern card showing seven dots, and Adam drew one with four dots. Asked how many dots they had together--without looking at the partner's card--they were not quite sure what to do. Adam made a first attempt of counting but concluded, "I don't know." Ben counted the dots on his card twice and then suggested:

Ben: Let's count a little aloud.
(To Adam) I'll start first, and then you go, you think what I said, what number I said, and then what's after that, all right, and then we will get it.
One, two, three, four, five, six, seven (counts his dots, then to Adam) eight--,

Adam: Eight, nine, ten, eleven, twelve (touches one dot twice, corrects himself), eight, nine, ten, eleven.
(To the teacher) Eleven.

Ben: Yeah, we did it.

Ben's suggestion could have resulted in two separate, unrelated counting episodes had Adam started his part of the counting activity with "one." Instead, Ben coordinated the two activities of counting the dots on each card into one counting

episode by regarding Adam's counting as an extension of his own. He anticipated prior to counting that the result of his own counting activity would determine the starting point for Adam's; he saw his and Adam's dots as belonging to one collection that was partitioned in a particular way.

The difficulty of this task did not result from the numerosity of the involved collections (seven and four dots). Both students had counted partly screened collections with either four or seven objects screened by making an intuitive extension of counting and keeping track of the counting acts in the extension with finger patterns. Instead, it was the different task format that caused some initial uncertainty; instead of having the whole collection--even with part of it covered--in their perceptual field, each student could see only the items on his own card. This change in task format and the resulting uncertainty seemed to be the starting point for Ben's initiation of teamwork. Getting familiar with the new task made further cooperation unnecessary; Ben and Adam's cooperative solution of the first task in Peek Through the Wall was the only incident of cooperation during the game. It was also the only example of cooperation involving a collection with less than 10 objects covered.

Size of covered collection as source of difficulty. Ben and Adam, as well as Andrea and Emily, had constructed finger patterns up to 10, and they usually activated those patterns to keep track of their counting acts in tasks with 10 or fewer items screened. But none of the four students had constructed finger patterns above 10. Consequently, to solve a task with more than 10 items covered, the students had to modify their schemes in some way. For the students of both pairs, the initial modification involved pooling fingers in order to establish a visual representation of the covered items. For both pairs, the situation leading to a sequence of cooperative counting episodes was a missing-item task with 14 of 17 checkers covered (see Figure 6). The move toward cooperation occurred after the initial counting activities had not led to a satisfactory solution. For each pair, I briefly describe the initial counting activities and then elaborate on the move toward cooperation.

Ben and Adam (Session 8) initially took turns. Adam, after extending all 10 fingers and then 4 fingers a second time, estimated 12 checkers under the cloth. He made this global judgment instead of looking back at the just completed activity to count how many fingers he had extended. Ben, after extending 10 fingers ("four, five, . . . , thirteen,"), folded down four fingers ("fourteen, fifteen, sixteen, seventeen."). In contrast to Adam, Ben set out to review how many fingers he had used. He slightly bent the fingers still extended ("One, two, . . . , six"), then extended the 4 previously folded fingers ("seven, eight, nine, ten,"), stopped and said in astonishment, "Ten!". He had a disconcerted look on his face. He then made an estimate of 13 checkers under the cloth. Like Adam, Ben had an intuitive feeling that more than 10 checkers were covered.

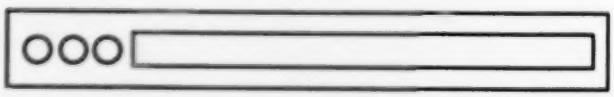

Figure 6. Missing-addend task: 17 checkers in all.

At this point, the teacher suggested to try again, but Ben initiated a cooperative venture instead:

> Ben: (Excited, to Adam) Hey, how about you count those three-- (points to the visible checkers), and I use my hand if you do-- want to do the other ones, all right? Then you can count them, and we'll know.

Ben's initiative led to a cooperative counting episode in which Adam first counted the three visible checkers ("One, two, three,"), then extended, one-by-one, his fingers to count the first 10 checkers under the cloth ("four, five, . . . , thirteen,"). At this point, Ben continued and extended four of his fingers ("fourteen, fifteen, sixteen, seventeen"), placing his hand beside Adam's two hands. After the boys set out to count how many fingers had been extended in all, they uncovered the checkers and found that they had indeed found how many checkers were under the cloth.

Ben's initiation of pooling fingers seemed to be the result of a conflict between his original result of "ten" and his intuitive feeling that there should be more than 10

checkers under the cloth. Because of this conflict, Ben's vague intuition of "more than ten" seemed to become more explicit. However, "more than ten" was still not explicit enough because it did not tell how many checkers were under the cloth. Ben knew that counting was a way to make "more than 10" more explicit ("Then you can count them, and we'll know.") Initiating the cooperative counting activity seemed to be more accessible to Ben than attempting to recount and to monitor his counting acts during the recounting activity. Monitoring his counting acts during the activity rather than retrospectively would have required a series of progressive uniting operations which were beyond the operations available to Ben in his figurative counting stage.

Emily and Andrea encountered the missing-added task (Figure 6) in Session 6. Their initiation of cooperation was a function of Andrea's more advanced counting stage (initial number sequence) and the girls' competitive friendship (Wiegel 1993). While Emily was still trying to understand what to do, Andrea was already counting subvocally. She extended all fingers of her right hand, then the fingers of her left hand, and then four fingers of her right hand again. She then exclaimed, "There's not enough for hands!" and reached for one of Emily's hands.

Like Ben and Adam, Andrea was aware that she had extended more than 10 fingers, and, like the boys, she did not represent and review her counting activity in order to make the indefinite awareness of "more than 10" definite. She also experienced a conflict, indicated by her remark, "There's not enough for hands!" In contrast to Ben, Andrea did not resolve this conflict by making an estimate. Because she was able to identify and verbalize the source of her problem ("not enough for hands"), she set out to solve her problem and reached for an additional hand. Andrea's more acute awareness of her actions was also apparent in her reaction to Emily's refusal to give assistance and her suggestion how to solve Andrea's problem:

Emily: (Pulls back, holds up both hands) You can go over on your hands again.

Andrea: I need one of your hands to help me.

Emily: No.
You can count over on your hands again.

Andrea: I know, but that's not what--
that's what not helping me.

Emily's refusal to lend a hand provides a glimpse at the girls' competitive friendship. It was important to them that they got equal attention from the teacher and that they were equally successful. At this point, Emily was still trying to understand what she was supposed to do whereas Andrea had already completed her first attempt at the task. It is possible that Emily was preoccupied with getting her own turn. In addition, Andrea did not announce or explain her intentions, nor did she--at first--ask for help. Emily's initial move of pulling back can also be interpreted as a reaction to being startled by Andrea's unexpected action.

Usually, the girls succeeded in resolving such a situation on their own--given time. In this case, however, the teacher intervened and suggested to start with Andrea's idea of sharing hands and then to try Emily's proposal of using the same hands over. The students followed the teacher's proposal only in part: Once they had counted how many objects were under cloth by pooling their fingers, they continued to do so and did not come back to Emily's idea. In the episode with Ben and Adam, the teacher's role in the actual move toward a cooperative activity was more subtle than her intervention with Emily and Andrea. When Ben gave voice to his insight to share hands, the students turned to the teacher--first Adam, then Ben--as if asking for permission. They took the teacher's response, "It's up to you," as the starting point for their activity.

Analysis of a Cooperative Counting Episode

Several cooperative counting episodes evolved in the context of counting the items in towers built from small (closed) cardboard boxes which the students had filled with beans. Ben and Adam's work with these towers illustrates the necessity for each student to complement a cooperative venture with an individual counting activity to

construct a solution for himself. To show these individual constructions within the cooperative activities, I present their work on the first tower in detail. The two boxes in the tower contained 13 and 10 beans.

The Initiation of Cooperation

Asked to count how many beans were in the two boxes, Ben seemed to sense that this task would be difficult and immediately suggested to pool fingers. In this instance, his move to cooperation did not arise out of frustration over the difficulty of a task but in anticipation of such a difficulty, and his initiation of teamwork might was probably triggered by his records of experience of the successful cooperative work during the previous session (see pp. 19-20).

Ben: Let's count together, 'cause it might be a little–
I count first, all right and, and if there's not enough you count with your hands, all right?

Adam: Okay--

Ben: 'Cause we gonna do teamwork.
(Sequentially extends five fingers of his left hand) One, two, three, four, five--

Adam: (Joins subvocally) Four, five--

Ben: (Opens the right hand in one movement, wiggles the fingers.) There's ten, all right.

Adam: I know--

Ben: Ten-- (looks at Adam),

Adam: (Looks into the air, then at his right hand, sequentially extends his index, middle, and ring finger) eleven, twelve, thirteen.

Ben: (With Adam) eleven, twelve, thirteen.

Adam: (Moves his right hand--three fingers extended--besides Ben's 10 fingers) okay--

Adam needed some time to step back from his initial intention to take turns ("Let me count first,") and to tune into his partner's activity, especially because Ben did not make any further comments about his intentions. When Ben started to count, Adam looked down, his head in his hands, a frown on his face, but then subvocally joined Ben's counting acts ("Four, five"). He seemed to need Ben's prompt of "Ten" before he continued the counting activity. Adam's movement of his extended fingers to the side of Ben's hands established his involvement in the cooperative activity.

Adam's Construction of a Solution

Adam intensified his involvement when he took charge and counted the 13 extended fingers, and he did so twice. Both times he counted his own three extended fingers first, then the fingers on Ben's two hands. He closed this part of the episode with "Okay, we got thirteen all set up," then reached for the other box:

Adam: Thirteen--

Ben: And ten more.

Adam: Okay, thirteen, what comes after thirteen, fourteen?
Fourteen-- (extends his right index finger).

At this point Ben tried to take over; he grabbed Adam's hand so he would put up his three fingers again. But Adam withdrew and continued:

Adam: Fourteen, fifteen, sixteen, . . ., twenty-three (sequentially extends his 10 fingers).
Twenty-three altogether.
(Briefly presents the ten fingers, then takes the two boxes into his hands.) All these,
[put] together.

The way Adam emphasized "thirteen" in his continuation of counting strongly suggests that he applied the uniting operation to the 13 previously counted fingers. In this context, the number word "thirteen" referred to an experiential composite unit whose content was the counted fingers (Steffe & Cobb, 1988, p. 77). Adam's determination to resist Ben's interference and to continue counting emphasizes that his activity was purposeful and that he knew what he was going to do before he was doing it, namely to perform ten counting acts beyond "thirteen."

When Adam took charge, Ben stayed involved. However, there is no indication that Ben also applied the uniting operation to make an experiential composite unit of 13. Consequently, he could not understand what Adam was doing when he counted 10 more past 13, keeping track with the same fingers that had been part of the representation of 13. Instead, Ben tried to reinstate the representation for 13. When Adam continued with his actions (counting 10 beyond 13), Ben turned towards him and watched. And although he subvocally joined his partner in saying the number

words, he did not accept Adam's conclusion that there would be "twenty-three all together."

The Second Organization of Team Effort and Ben's Construction of a Solution

Ben immediately started a new counting activity. He first counted his own fingers ("one, two, . . ., ten"), then Adam's ("eleven, twelve, . . ., twenty"), and finally two of the teacher's ("twenty-one, twenty-two"). Trying to justify his solution by counting again, he seemed to experience cognitive conflict; he touched one of his hands, then one of the teacher's, then turned to Adam and took his hand. When the teacher noted, "I don't understand what you are doing, Ben," he initiated a new team effort:

> Ben: Ah-- (takes his head into both hands), I need to get ten in my head, then count my tens here (puts his hands on the table, palms up, all fingers extended), then you count thirteen, you guys, you two make thirteen.

Here the teacher's request for clarification seemed to be critical for Ben's organization of the situation. The necessity to give an answer, that is, to verbalize his intentions, helped him clarify for himself what he was trying to do. Asked how to "make thirteen," Ben explained:

> Ben: Well make thir--, put up three here, put three (takes three of Adam's fingers and extends them), and this is thirteen (puts his two open hands besides Adam's three extended fingers.
> (To the teacher) You have to-- put your two hands out.
> (To Adam) And you put up three, and now it's--
> (Counts the extended fingers, including his.)

Ben did not have to count in order to show how to "make thirteen." Instead, he regenerated the result of the prior cooperative counting experience by first asking for three of Adam's fingers and then placing his two hands besides those three extended fingers. This regeneration was not a copy of the earlier representation of 13 because Ben reversed the order of the two components 10 (two full hands) and 3 (three fingers). Whereas Adam had applied the uniting operation to the 13 counted fingers and made an experiential unit of 13, Ben focused on his 10 and Adam's 3 fingers separately. He applied the uniting operation sequentially and made experiential composite units of 10 and 3 that he then combined to make 13. His ability to see 13 as

10 and 3 as well as 3 and 10 and his flexibility in exchanging the teacher's hands for his indicate that he was aware of the finger patterns as one entity rather than of only the individual fingers (see Steffe & Cobb, 1988, p. 158).

Consolidation

Adam watched closely as Ben counted the 23 extended fingers. As soon as Ben had finished, Adam took the initiative and started again. This new initiative eventually led to a counting activity in which both partners took part. Adam started, "One, two, . . . , ten," extending his fingers, then Ben extended three of the teacher's fingers, "eleven, twelve, thirteen," and finally both counted in synchrony "fourteen, fifteen, . . . , twenty-three," while Ben kept track with his fingers. After this activity, executed in collaboration, both students seemed to experience a sense of closure and were ready to leave this particular task.

Reflection on Cooperation

Some of the striking features of the described cooperative counting episode were the autonomy and selfdirectedness of Ben and Adam. From the beginning of the episode, both students took charge of the task and made it their own. Adam's immediate engagement was evident in a spontaneous completion of the teacher's introduction ("How many are there together, right?") and in his claim to start ("Let me count first,"). Ben's involvement was visible in his focus on the size of the to-be-counted collections and his subsequent initiation of team work. The only contribution of the teacher at this point consisted of the selection of the boxes and in the initial statement of the task.

The students stayed actively involved and in charge throughout the episode. Their actions such as their participation in the partner's counting, the initiation of new counting activities, and the conclusion of the episode at the point of closure were selfdirected and seemed to be independent of the presence of a teacher. Apart from the teacher's clarifying question to Ben and some technical help (i.e., wiggling her thumb Ben had missed), her role in the episode was that of a participant rather than

that of a leader. Indeed, the students, in particular Ben, seemed to take the teacher's participation in the cooperative solution process for granted ("You guys, you two make thirteen."). In taking responsibility and initiative for solving a situation problematic for him, Ben created a community of equal participants who were contributing--from his perspective--toward the solution of that problematic situation. Ben's initiative evolved into an situation of learning for both boys in their attempts to solve the problematic situation (Lo, Wheatley, & Smith, 1994).

Emily and Andrea, in their cooperative counting episodes, were usually not as focused toward a common goal as Ben and Adam. The girls' competitive friendship required more explicit negotiations as to who would start, how they would work together, and when and how to share fingers. And although they too did not hesitiate to use the teacher's fingers, they did not take the inclusion of the teacher into a community of equal participants as self-evident as Ben seemed to do

An additional reason for the different structure of cooperative episodes with Emily and Andrea was that the girls were not as closely matched with respect to their development of counting as were Adam and Ben. Andrea could count on whereas Emily was just in the process of constructing it. Many of the tasks challenging for Ben, Adam, Emily and requiring a cooperative approach, Andrea could solve by herself. Consequently, some of the cooperative episodes with Andrea and Emily started out as situations of helping: Andrea was the helper and Emily was the partner who needed help and who asked for the help. Out of these situations of helping, the girls would then develop a cooperative activity in which they worked together and in which their counting acts merged into a single counting activity.

Summary and Discussion

During the course of the teaching sessions, kindergarten students working in pairs generated four strategies to organize their counting in joint counting tasks: counting

side by side, counting at the same time, taking turns, and working cooperatively. The four strategies required different levels of mutual coordination of actions, with counting side by side needing the least and cooperation needing the most coordination. When taking turns, the students' involvement in the partner's turn included watching, supporting the partner, watching and joining, helping, and completing the partner's task. As the teaching sessions progressed, all students made at least some progress toward forms of organization requiring more coordination and toward increased involvement in the partner's actions. The students of two pairs solved some counting tasks in cooperation. Cooperative counting episodes evolved as result of the teacher's instruction to work as a team or when the students encountered a task they could--at the time--not solve alone. The students who solved counting tasks in cooperation were the those with the most advanced counting schemes; they had constructed or were close to constructing the initial number sequence. This apparent relationship between the students' counting stage and their ability to work cooperatively seems reasonable if one considers some of the conditions of cooperation.

Working cooperatively requires an act of planning to establish a common goal and to project an initial action, and thus reflection on past experiences and anticipation of future ones. Reflection and anticipation are two attributes characteristic of students having constructed the initial number sequence and usually are not available to perceptual children within their counting activities. For perceptual children, most number words refer to "the transitory [emphasis added] experience of counting collections" (Steffe, 1992, p. 87), and no object concepts are associated with the number words. That is, students with only perceptual counting schemes need to count in order to give meaning to most number words. Students who have constructed the initial number sequence, on the other hand, do not have to execute a counting activity; a number word can symbolize and stand for the activity itself as well as for the result of the activity (e.g., counting on). The ability to take a counting activity as given is the

result of reflection on past counting experiences. This ability is paramount in a cooperative counting episode if the counting acts of both partners are to merge into a single activity. In addition, students working cooperatively need to make sense of the partner's actions. They need to be able to interpret the partner's activity and to relate this interpretation to their own experience, and these mutual interpretations have to be compatible. Again, the interpretation of the partner's actions in relation to the student's own actions requires reflection, the ability to stand back and look on one's actions from a distance.

The suggested connection between the ability to work cooperatively and the students' counting stage is in accord with Doise and Mugny's (1984) proposition of a close and mutually reciprocal relationship between social interaction and cognitive development. In this relationship, "interaction enables the individual to master certain abilities which allow him to participate in more complex social interactions which in turn promotes continued cognitive development" (p. 23). A second manifestation of the mutually reciprocal relationship between social interaction and cognitive development can be seen in the activity I termed watching and joining.

Watching and joining was defined as a form of participation in which the noncounting student joined the partner's utterances of number words while the partner, pointing to or displacing the objects, was in charge of the counting activity. The action of watching and joining has significance for promoting more sophisticated forms of social interaction as well as for fostering cognitive development. Watching and joining is relevant for cognitive development in that it is an opportunity for the student to look at his or her "own actions from the outside and from the inside . . . . This may facilitate reflection on the action as an object of thought." (Sinclair, 1990, p. 22). Watching and joining can also provide a basis for the student's ability to take the partner's counting activity as given, that is, to substitute the partner's counting for his or her own activity. One of the perceptual student's continuation of his partner's counting

activity (see pp. 12-13) illustrates the power of watching and joining for his cognitive growth.

With respect to social interaction, watching and joining can be a stepping stone toward cooperation. In watching and joining, the student is tuning in to his or her partner's counting activity. Watching the partner count provides an opportunity to relate to the partner's activity and to interpret this activity in reference to the student's own counting experiences. Uttering number words in synchrony with the partner brings the partner's activity even closer to the student's own experiences. Watching and joining provides a frame for "taken-to-be-shared" (Wood, Cobb, Yackel, & Wheatley, 1990, p. 5) experience and knowledge.

In general, the students' mutual involvement in the partner's counting activities during taking turns can be seen as preparation for more sophisticated social interactions such as cooperative solutions. As the students worked together, some cooperative behaviors identified by Cohen (1986), such as helping, listening, and being responsive to the needs of the partner, evolved implicitly, within the students' counting activities. Of special significance for the development of such cooperative behaviors were the tasks involving unscreened collections. For the students in the perceptual counting stage and those in transition to the figurative stage, these tasks were in the their "comfort zone" (Olive, personal communication, 1993) but still challenging enough to be interesting. As observers, the students could relate to his or her partner's counting activity and interpret it with respect to their own experience. These interpretations seemed to be mutually compatible and led, in turn, to increased and more sophisticated participation.

In conclusion, the results of the study suggest that working in pairs can be valuable for kindergarten students. In contrast to Wilkinson (1988), who limited the role of small-group work in kindergarten to one of preparation for within-class instructional grouping, I see more profound benefits of the pair-working arrangement. Working in pairs can lead to learning opportunities different from those occurring in a whole-class

setting or in one-to-one teaching situations. Working in pairs can support and enhance the students' cognitive development and can promote ever more sophisticated ways of social interaction. Students in the perceptual or figurative counting stage can devise forms of participation in which they relate the partner's activity to their own counting experiences. If students are able to reflect on and anticipate their actions, working in pairs can lead to cooperative solutions. With respect to counting, students in the process of constructing the initial number sequence can take their counting activity as an object of thought and have the potential to engage in cooperative counting episodes. However, a student's counting stage is only one factor influencing the quality and success of working and collaborating with a partner. Other mediating factors such as the students' motivation and attitude, were outside the focus of this paper.

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